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Higher order contributions to the effective action of $\mathcal{N}=4$ super Yang-Mills

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Abstract: The one-loop low-energy effective action for non-abelian $\mathcal{N}=4$ supersymmetric Yang-Mills theory is computed to order F^6 by use of heat kernel techniques in $\mathcal{N}=1$ superspace. At the component level, the F^5 terms are found to be consistent with the form of the non-abelian Born-Infeld action computed to this order by superstring methods. The F^6 terms will be of importance for comparison with superstring calculations.

Keywords: D-branes, Supersymmetric Effective Theories, Superspaces.

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Contents

1. Introduction

There is considerable interest in the issue of deformations of maximally supersymmetric Yang-Mills theories [1]–[6]; for a review see [7]. Such deformations arise in the context of superstring theories, where the low-energy effective actions for D-branes admit expansions in powers of the string tension[1] α'. The lowest order term is a maximally supersymmetric Yang-Mills action. For a single D-brane, the terms in the low-energy effective action which do not contain derivatives of the field strength are known to all orders in α': they are given by the Born-Infeld action [8, 9]. In the case when there are N coincident D-branes, the resulting low-energy effective action has been dubbed the non-abelian Born-Infeld action [10], because the lowest order term in the expansion is the action for $\mathrm{SU}(N)$ supersymmetric Yang-Mills theory [11], and it reduces to the Born-Infeld action in the abelian case. Due to the Bianchi identity

$$[F_{ab}, F_{cd}] = 2\mathrm{i}\, \nabla_{[a} \nabla_{b]} F_{cd}\,, \tag{1.1}$$

[1]The generic structure is of the form $\sum_{n=0}^{\infty} c_n\, (\alpha')^n\, F^{n+02}$, where F^n denotes terms of mass dimension $2n$ in F and its covariant derivatives. For D-brane probes in the background of a stack of D-branes, the expansion parameter is not α', but is determined by the vacuum expectation values of scalar fields which specify the separation of the probe from the stack.

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it is not possible to consistently truncate the non-abelian Born-Infeld action to constant field strength, and so derivative corrections must be considered [7]. As yet, only a few terms in the α' expansion of the non-abelian Born-Infeld action are known.

Of particular interest is the question as to whether supersymmetry is a sufficiently strong constraint to uniquely specify the form (up to field redefinitions) of the deformation of a maximally supersymmetric Yang-Mills theory [4, 5]. If this were the case, then any means to compute a supersymmetric deformation would yield the non-abelian Born-Infeld action. In particular, low energy effective actions for supersymmetric Yang-Mills theories would be related to the non-abelian Born-Infeld action. However, since the effective action is dependent on the choice of gauge, with a change of gauge inducing a field redefinition, direct comparison of low-energy effective actions with deformations obtained by other means is potentially non-trivial.

The F^5 contributions to the non-abelian Born-Infeld action have recently been calculated in full by several different methods [12, 13, 14]. In [12], the one-loop low-energy effective action for $\mathcal{N}=4$ supersymmetric Yang-Mills theory in four dimensions was calculated using supergraphs, and the result was:

$$\begin{aligned}\kappa_1 \operatorname{Tr} \Big(& 2(\nabla^e F^{ab})(\nabla_e F_{bc})F^{cd}F_{da} + 2(\nabla^e F^{ab})(\nabla_e F^{cd})F_{bc}F_{da} + (\nabla^e F^{ab})(\nabla_e F_{ca})F_{bd}F^{dc} - \\ & - \frac{1}{2}(\nabla^e F^{ab})(\nabla_e F_{ab})F^{cd}F_{cd} - \frac{1}{2}(\nabla^e F^{ab})(\nabla_e F^{cd})F_{ab}F_{cd} - \\ & - \frac{1}{2}(\nabla^e F^{ab})(\nabla_e F^{cd})F_{cd}F_{ab} + 2\mathrm{i}F^{ab}F_{bc}F^{cd}F_{de}F^{e}{}_{a} + 7\mathrm{i}F^{ab}F_{bc}F^{cd}F^{e}{}_{a}F_{de} + \\ & + 3\mathrm{i}F^{ab}F^{cd}F^{e}{}_{a}F_{bc}F_{de} - 4\mathrm{i}F^{ab}F^{cd}F_{bc}F^{e}{}_{a}F_{de} \Big) , \end{aligned} \tag{1.2}$$

where κ_1 is a normalization. On the other hand, Koerber and Sevrin [13] used an approach based on the requirement that certain BPS solutions should exist to the equations of motion derived from the non-abelian D-brane effective action, extending earlier use of this method for the abelian case [15]. At order $(\alpha')^3$, this approach yields[2]

$$\begin{aligned}\kappa_2 \operatorname{Tr} \Big(& F^{ab}(\nabla_a F^{cd})(\nabla^e F_{bc})F_{de} - (\nabla^e F^{ab})F^{cd}(\nabla_a F_{bc})F_{de} - \frac{1}{2}(\nabla^e F^{ab})(\nabla_e F_{ca})F_{bd}F^{dc} - \\ & - \frac{1}{2}(\nabla^e F^{ab})F_{da}(\nabla_e F_{bc})F^{cd} + \frac{1}{8}(\nabla^e F^{ab})F^{cd}(\nabla_e F_{ab})F_{cd} - \\ & - \frac{\mathrm{i}}{10}F^{ab}F_{bc}F^{cd}F_{de}F^{e}{}_{a} - \frac{\mathrm{i}}{2}F^{ab}F_{bc}F^{cd}F^{e}{}_{a}F_{de} - \\ & - \frac{\mathrm{i}}{2}F^{ab}F^{cd}F_{bc}F^{e}{}_{a}F_{de} + \frac{\mathrm{i}}{7}F^{ab}F^{cd}F^{e}{}_{a}F_{bc}F_{de} \Big) , \end{aligned} \tag{1.3}$$

where κ_2 is a normalization constant. Comparison of (1.2) and (1.3) is not straightforward

[2]Partial results at order $(\alpha')^3$ had previously been obtained in [16, 17, 18]. F^5 terms in ten dimensional super Yang-Mills were given in [19].

due to the identity (1.1). However, when written using the basis for the various tensor structures adopted in (1.3), the result (1.2) takes the form

$$
\begin{aligned}
8\kappa_1 \,\mathrm{Tr}\Big(& F^{ab}(\nabla_a F^{cd})(\nabla^e F_{bc})F_{de} - (\nabla^e F^{ab})F^{cd}(\nabla_a F_{bc})F_{de} - \frac{1}{2}(\nabla^e F^{ab})(\nabla_e F_{ca})F_{bd}F^{dc} - \\
& - \frac{1}{2}(\nabla^e F^{ab})F_{da}(\nabla_e F_{bc})F^{cd} + \frac{1}{8}(\nabla^e F^{ab})F^{cd}(\nabla_e F_{ab})F_{cd} + \\
& + \frac{3\mathrm{i}}{20}F^{ab}F_{bc}F^{cd}F_{de}F^{e}{}_{a} + \frac{\mathrm{i}}{8}F^{ab}F_{bc}F^{cd}F^{e}{}_{a}F_{de} - \\
& - \frac{\mathrm{i}}{2}F^{ab}F^{cd}F_{bc}F^{e}{}_{a}F_{de} + \frac{33\mathrm{i}}{40}F^{ab}F^{cd}F^{e}{}_{a}F_{bc}F_{de}\Big) \,.
\end{aligned} \tag{1.4}
$$

As can be seen, the terms containing covariant derivatives of the field strength coincide with (1.3), but the terms without covariant derivatives differ [13, 20].

A number of tests have successfully been applied to confirm that expression (1.3) is consistent with string theoretic predictions [20, 21]. Most recently, a string theory calculation of the full five-point scattering amplitude for gluons has been carried out [14], from which it is inferred that the corresponding low-energy effective action has precisely the order $(\alpha')^3$ terms (1.3). This technique was first applied at order $(\alpha')^2$ by Gross and Witten [22]. Other approaches have also provided information on the Born-Infeld action at this order [23, 24, 25].

In this paper, the one-loop low-energy effective action for non-abelian $\mathcal{N}=4$ supersymmetric Yang-Mills theory is calculated in $\mathcal{N}=1$ superfield form through to order F^6. The technique employed is a modification of that developed in [26, 27] based on the properties of "moments" of heat kernels.[3] At order F^5, extraction of components from the resulting superfield expression yields (1.3), rather than (1.2). The fact that three different means to calculate a F^5 deformation of supersymmetric Yang-Mills theory yield the same result (1.3) is evidence for the existence of a unique deformation at this order. If indeed this is the case, it also suggests that the F^5 terms in the low-energy effective action for $\mathcal{N}=4$ supersymmetric Yang-Mills theory are not renormalized beyond one-loop. The one-loop non-abelian F^6 terms computed in this paper are potentially important for comparison with recent string theoretic results [30, 31, 32], as are the recently computed two-loop abelian F^6 terms [33].

The paper is organized as follows. Firstly the quantization of non-abelian $\mathcal{N}=4$ supersymmetric Yang-Mills theory in $\mathcal{N}=1$ superfield form is briefly reviewed, including the background field method, heat kernels and zeta function regularization. In section 3 a general expression adapted to the asymptotic expansion of the heat kernel is derived. This expression is then used in section 4 to compute the F^5 terms in the low-energy effective action in superfield form, from which we extract the bosonic component and make comparisons with the results (1.2) and (1.3). In section 5 we explain how the technique generalizes to allow a computation of the F^6 terms in the one-loop low-energy effective action, and the full superfield result can be found in the appendix B. Some comments on the form of the result are included in section 6.

[3] For an alternative technique, see [28, 29].

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We adopt the conventions and notation of [34, 35].

2. Quantization of $\mathcal{N}=4$ super Yang-Mills

The non-abelian $\mathcal{N}=4$ supersymmetric Yang-Mills action cast in $\mathcal{N}=1$ superfield form is

$$S = \frac{1}{g^2}\,\mathrm{Tr}\left(\int \mathrm{d}^8 z e^{-2V}\bar{\Phi}_i e^{2V}\Phi_i + \frac{1}{4}\int \mathrm{d}^6 z W^\alpha W_\alpha + \frac{1}{4}\int \mathrm{d}^6\bar{z}\bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha} + \right.$$
$$\left. + \frac{\sqrt{2}}{3!}\int \mathrm{d}^6 z\epsilon^{ijk}[\Phi_i,\Phi_j]\Phi_k + \frac{\sqrt{2}}{3!}\int \mathrm{d}^6\bar{z}\epsilon^{ijk}[\bar{\Phi}_i,\bar{\Phi}_j]\bar{\Phi}_k\right), \tag{2.1}$$

where

$$W_\alpha = -\frac{1}{8}\bar{D}^2\left(e^{-2V}D_\alpha e^{2V}\right) \tag{2.2}$$

and Φ_i ($i = 1, 2, 3$) are chiral superfields. All superfields are Lie-algebra valued, for example $\Phi_i = \Phi_i^I T^I$, with the Hermitian generators T^I satisfying:

$$\left[T^I, T^J\right] = \mathrm{i} f^{IJK} T^K\,. \tag{2.3}$$

In the $\mathcal{N}=1$ background field formalism for supersymmetric non-abelian gauge theories [36], it is necessary to introduce a non-linear background-quantum splitting to ensure a gauge invariant effective action. Defining

$$e^{2V} \equiv e^w e^{\bar{w}}\,, \tag{2.4}$$

the background-quantum splitting is given by

$$e^w = e^{w_B} e^{w_Q}\,, \tag{2.5}$$

or equivalently

$$e^{2V} = e^{w_B} e^{2V_Q} e^{\bar{w}_B}\,, \tag{2.6}$$

where the subscripts B and Q denote background and quantum pieces, respectively. Since we will be performing a one-loop calculation it is only necessary to retain terms quadratic in quantum fields in the action. After background covariant gauge fixing [36, 37], the quantum quadratic action for $\mathcal{N}=4$ super Yang-Mills becomes

$$S_{\mathrm{quad}} = \int \mathrm{d}^8 z V_Q(\Delta_1 - m^2)V_Q\,, \tag{2.7}$$

where

$$\Delta_1 = \mathcal{D}^a\mathcal{D}_a - W_B^\alpha\mathcal{D}_\alpha - \bar{W}_B^{\dot\alpha}\bar{\mathcal{D}}_{\dot\alpha}\,. \tag{2.8}$$

The mass $m^2 = |\Phi_B|^2$ is introduced via a constant chiral scalar superfield background Φ_B, and the low-energy effective action has an expansion in inverse powers of m^2. The W_B's and $\mathcal{D}$'s are background superfield strengths and background gauge covariant derivatives, respectively, defined and related by (dropping all B subscripts):

$$\begin{aligned}\mathcal{D}_\alpha &= e^{-w} D_\alpha e^{w}\,, \qquad \bar{\mathcal{D}}_{\dot\alpha} = e^{\bar w}\bar D_{\dot\alpha} e^{-\bar w}\,,\\ \mathcal{D}_a &= -\frac{1}{2}(\tilde\sigma_a)^{\dot\alpha\alpha}\mathcal{D}_{\alpha\dot\alpha} = -\frac{\mathrm{i}}{4}(\tilde\sigma_a)^{\dot\alpha\alpha}\left\{\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot\alpha}\right\},\\ [\bar{\mathcal{D}}_{\dot\alpha},\mathcal{D}_{\beta\dot\beta}] &= 2\mathrm{i}\varepsilon_{\dot\alpha\dot\beta}W_\beta\,, \qquad [\mathcal{D}_\alpha,\mathcal{D}_{\beta\dot\beta}] = 2\mathrm{i}\varepsilon_{\alpha\beta}\bar W_{\dot\beta}\,.\end{aligned} \tag{2.9}$$

One can efficiently calculate the one-loop effective action through zeta function regularization, and it is just $-\frac{1}{2}\zeta'(0)$, where the zeta function is defined by

$$\zeta(s) = \frac{\mu^{2s}}{\Gamma(s)}\int_0^\infty \mathrm{d}t\, t^{s-1} e^{-tm^2} K(t)\,. \tag{2.10}$$

In this expression μ is the renormalization point and $K(t)$ is the functional trace of the heat kernel associated with the operator Δ_1,

$$K(t) = \mathrm{Tr}\int \mathrm{d}^8 z \lim_{z'\to z} e^{t\Delta_1}\delta^{(8)}(z,z') \equiv \mathrm{Tr}\int \mathrm{d}^8 z \lim_{z'\to z} K\left(z,z',t\right). \tag{2.11}$$

Here Tr denotes the trace over gauge indices and $\delta^{(8)}(z,z')$ is the superspace delta function,

$$\delta^{(8)}(z,z') = \delta^{(4)}(x,x')\delta^{(2)}(\theta-\theta')\delta^{(2)}\left(\bar\theta-\bar\theta'\right). \tag{2.12}$$

Introducing a plane wave basis for the delta functions,

$$\delta^{(4)}(x,x') = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} e^{\mathrm{i}k^a\omega_a}\,, \tag{2.13}$$

$$\delta^{(2)}(\theta-\theta') = 4\int \mathrm{d}^2\epsilon\, e^{\mathrm{i}\epsilon^\alpha(\theta-\theta')_\alpha}\,, \qquad \delta^{(2)}(\bar\theta-\bar\theta') = 4\int \mathrm{d}^2\bar\epsilon\, e^{\mathrm{i}\bar\epsilon_{\dot\alpha}(\bar\theta-\bar\theta')^{\dot\alpha}}\,, \tag{2.14}$$

where

$$\omega_a = x_a - x'_a - i\theta\sigma_a\bar\theta' + i\theta'\sigma_a\bar\theta\,, \tag{2.15}$$

and defining

$$\int \mathrm{d}\eta = 16\int \frac{\mathrm{d}^4 k}{(2\pi)^4}\int \mathrm{d}^2\epsilon\int \mathrm{d}^2\bar\epsilon\,, \tag{2.16}$$

$K(z,z',t)$ becomes

$$K(z,z',t) = \int \mathrm{d}\eta\; e^{\mathrm{i}k^a\omega_a} e^{\mathrm{i}\epsilon^\alpha(\theta-\theta')_\alpha} e^{\mathrm{i}\bar\epsilon_{\dot\alpha}(\bar\theta-\bar\theta')^{\dot\alpha}}\, e^{t\Delta} \tag{2.17}$$

with

$$\Delta = X^a X_a - W^\alpha X_\alpha - \bar W^{\dot\alpha}\bar X_{\dot\alpha}\,. \tag{2.18}$$

Here, the X's are defined by

$$\begin{aligned} X_a &= \mathcal{D}_a + ik_a\,,\\ X_\alpha &= \mathcal{D}_\alpha + i\epsilon_\alpha - k_{\alpha\dot\alpha}(\bar\theta-\bar\theta')^{\dot\alpha}\,,\\ \bar X_{\dot\alpha} &= \bar{\mathcal{D}}_{\dot\alpha} + i\bar\epsilon_{\dot\alpha} + k_{\alpha\dot\alpha}(\theta-\theta')^\alpha\,,\end{aligned} \tag{2.19}$$

and satisfy the algebra

$$
\begin{aligned}
\{X_\alpha, X_\beta\} &= \{\bar{X}_{\dot\alpha}, \bar{X}_{\dot\beta}\} = 0\,, \qquad \{X_\alpha, \bar{X}_{\dot\alpha}\} = -2iX_{\alpha\dot\alpha}\,, \qquad [X_a, X_b] = G_{ab}\,,\\
[X_\alpha, X_{\beta\dot\beta}] &= 2i\varepsilon_{\alpha\beta}\bar{W}_{\dot\beta}\,, \qquad [\bar{X}_{\dot\alpha}, X_{\beta\dot\beta}] = 2i\varepsilon_{\dot\alpha\dot\beta}W_\beta\,,\\
G_{\alpha\dot\alpha,\beta\dot\beta} &= (\sigma^a)_{\alpha\dot\alpha}(\sigma^b)_{\beta\dot\beta}G_{ab} = -\varepsilon_{\alpha\beta}(\bar{\mathcal{D}}_{\dot\alpha}\bar{W}_{\dot\beta}) - \varepsilon_{\dot\alpha\dot\beta}(\mathcal{D}_\alpha W_\beta)\,.
\end{aligned}
\tag{2.20}
$$

Taking the limit[4] $z' \to z$ in (2.17), one obtains

$$
K(z,t) \equiv \lim_{z'\to z} K(z,z',t) = \int \mathrm{d}\eta\, e^{t\Delta}\,. \tag{2.21}
$$

The kernel $K(z,t)$ has an asymptotic expansion in t in the limit $t \to 0$, and the leading term in the expansion is of order t^2. This fact can be seen by making the rescaling $k_a \to t^{-1/2}k_a$, and by observing that the integral over the fermionic parameters ϵ_α and $\bar\epsilon_{\dot\alpha}$ will bring down at least four factors of t. Defining the DeWitt-Seeley coefficients a_n in the usual manner,

$$
K(z,t) = \frac{1}{16\pi^2 t^2}\sum_{n=0}^{\infty} t^n a_n\,, \qquad a_i = 0\,, \qquad i = 0,1,2,3\,, \tag{2.22}
$$

the one-loop effective action then takes the form

$$
\Gamma_{(1)} \equiv -\frac{1}{2}\zeta'(0) = -\frac{1}{32\pi^2}\sum_{n=4}^{\infty}\frac{(n-3)!}{m^{2n-4}}\int \mathrm{d}^8 z\,\mathrm{Tr}(a_n)\,, \tag{2.23}
$$

which is an expansion in inverse powers of the mass parameter. At the component level, the non-trivial DeWitt-Seeley coefficients, a_n for $n \geq 4$, contain bosonic field strength terms of the form F^n. The first non-trivial coefficient, a_4, is well-known (see for example [38, 39]):

$$
\mathrm{Tr}(a_4) = \frac{1}{3}\,\mathrm{Tr}\left(2W^2\bar{W}^2 - W^\alpha\bar{W}_{\dot\alpha}W_\alpha\bar{W}^{\dot\alpha}\right). \tag{2.24}
$$

Our goal is to compute a_5 and a_6 in superfield form.

In general the process of asymptotically expanding heat kernels is involved and very laborious. The most direct route, expanding the exponential $e^{t\Delta}$, is cumbersome and really only practical for computing the first non-trivial DeWitt-Seeley coefficient. To calculate higher order coefficients it is necessary to introduce an efficient algorithm, which is done in the next section.

3. The differential equation approach

We proceed by modifying the differential equation approach developed in [26, 27]. Briefly, this approach involves generating a differential equation for $K(z,t)$. By exploiting certain properties of the kernel, and provided the background is sufficiently simple, the resulting equation can be solved either iteratively or in some cases exactly by expressing $dK(z,t)/dt$

[4] This limit is implicitly taken throughout the remainder of this paper.

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in terms of $K(z,t)$. For more complicated backgrounds, as with the case at hand, it becomes too difficult to express the differential equation in a form in which it can be solved.

However, the techniques employed in [26, 27] may be used in a rather different way to facilitate the computation of higher order DeWitt-Seeley coefficients in theories with arbitrary backgrounds. In this new approach, one does not actually attempt to solve the differential equation, as we now illustrate.

We begin by differentiating $K(z,t)$ with respect to t, which yields the differential equation

$$\frac{dK(z,t)}{dt} = K^a{}_a(z,t) - W^\alpha K_\alpha(z,t) - \bar{W}^{\dot\alpha} K_{\dot\alpha}(z,t)\,, \tag{3.1}$$

where the notation

$$K_{A_1 A_2 \dots A_n}(z,t) = \int \mathrm{d}\eta X_{A_1} X_{A_2} \dots X_{A_n} e^{t\Delta} \tag{3.2}$$

has been introduced, with the integration measure defined in (2.16). Using the identities

$$0 = \int \mathrm{d}\eta \frac{\partial}{\partial k_b}\left(X_a e^{t\Delta}\right) \tag{3.3}$$

and

$$\left[A, e^B\right] = \int_0^1 \mathrm{d}s\, e^{sB}[A,B]e^{(1-s)B}\,, \tag{3.4}$$

it follows that:

$$0 = \mathrm{i}\delta^b_a K(z,t) + 2\mathrm{i}t \int \mathrm{d}\eta\; X_a \sum_{n=0}^{\infty} \frac{t^n}{(n+1)!} ad^n_\Delta(J^b) e^{t\Delta}\,, \tag{3.5}$$

where

$$J^a = X^a - \frac{\mathrm{i}}{2} W\sigma^a\left(\bar\theta - \bar\theta'\right) - \frac{\mathrm{i}}{2}(\theta - \theta')\sigma^a \bar{W} \tag{3.6}$$

and ad^n denotes n nested commutators:

$$ad^0_A(B) = B\,, \qquad ad^n_A(B) = \left[A,\, ad^{n-1}_A(B)\right]. \tag{3.7}$$

After contraction of vector indices, this becomes

$$K^a{}_a(z,t) = -\frac{2}{t}K(z,t) - \int \mathrm{d}\eta X_a \sum_{n=1}^{\infty} \frac{t^n}{(n+1)!} ad^n_\Delta(J^a) e^{t\Delta}\,. \tag{3.8}$$

Similarly, using

$$0 = \int \mathrm{d}\eta\; \frac{\partial}{\partial \epsilon_\beta}\left(X_\alpha e^{t\Delta}\right) \tag{3.9}$$

and

$$0 = \int \mathrm{d}\eta\; \frac{\partial}{\partial \bar\epsilon_{\dot\beta}}\left(\bar X_{\dot\alpha} e^{t\Delta}\right) \tag{3.10}$$

it follows that:

$$W^\alpha K_\alpha(z,t) = -\frac{2}{t}K(z,t) + (\mathcal{D}_\alpha W^\alpha)K(z,t) + \int \mathrm{d}\eta X_\alpha \sum_{n=1}^{\infty} \frac{t^n}{(n+1)!} ad^n_\Delta(W^\alpha) e^{t\Delta} \tag{3.11}$$

and

$$\bar{W}^{\dot\alpha}K_{\dot\alpha}(z,t)=-\frac{2}{t}K(z,t)+(\bar{\mathcal{D}}_{\dot\alpha}\bar{W}^{\dot\alpha})K(z,t)+\int \mathrm{d}\eta\bar{X}_{\dot\alpha}\sum_{n=1}^{\infty}\frac{t^n}{(n+1)!}ad^n_{\Delta}(\bar{W}^{\dot\alpha})e^{t\Delta}\,, \tag{3.12}$$

respectively.

Finally, inserting (3.8),(3.11) and (3.12) into the differential eq. (3.1) one obtains:

$$\begin{aligned}\frac{dK(z,t)}{dt}-\frac{2}{t}K(z,t) &= -\int d\eta X_a\sum_{n=1}^{\infty}\frac{t^n}{(n+1)!}ad^n_{\Delta}(J^a)e^{t\Delta}-\\ &-\int d\eta\, X_\alpha\sum_{n=1}^{\infty}\frac{t^n}{(n+1)!}ad^n_{\Delta}(W^\alpha)e^{t\Delta}-\\ &-\int d\eta\bar{X}_{\dot\alpha}\sum_{n=1}^{\infty}\frac{t^n}{(n+1)!}ad^n_{\Delta}(\bar{W}^{\dot\alpha})e^{t\Delta}\,,\end{aligned} \tag{3.13}$$

where the $K(z,t)$ pieces have been brought to the left hand side and the Bianchi identity, $\mathcal{D}^\alpha W_\alpha=\bar{\mathcal{D}}_{\dot\alpha}\bar{W}^{\dot\alpha}$, has been used. The significance of this expression is seen in terms of the asymptotic expansion (2.22), where the left hand side is

$$\frac{dK(z,t)}{dt}-\frac{2}{t}K(z,t)=\frac{1}{16\pi^2}\sum_{n=0}^{\infty}(n-4)t^{n-3}a_n=\frac{a_5t^2}{16\pi^2}+\frac{2a_6t^3}{16\pi^2}+\cdots\,. \tag{3.14}$$

It is clear that in this particular combination of the kernel and its derivative, the first non-trivial coefficient, a_4, is absent.[5] Exploiting this fact, the objective now becomes to determine the DeWitt-Seeley coefficients by expanding the right hand side of (3.13) in a power series in t, and identifying it with the right hand side of (3.14).

The background has been arbitrary to this point. However, from now on it will be placed on-shell, $\mathcal{D}^\alpha W_\alpha=\bar{\mathcal{D}}_{\dot\alpha}\bar{W}^{\dot\alpha}=0$, as we are only interested in the on-shell effective action. Since the summation on the right hand side of (3.13) involves the repetitive calculation of commutators, it is first useful to establish the following relations:

$$\begin{aligned}[\Delta,X_a] &= 2G^b{}_aX_b+(\mathcal{D}_aW^\alpha)X_\alpha+(\mathcal{D}_a\bar{W}^{\dot\alpha})\bar{X}_{\dot\alpha}\,,\\ [\Delta,X_\alpha] &= (\mathcal{D}_\alpha W^\beta)X_\beta\,,\\ [\Delta,\bar{X}_{\dot\alpha}] &= (\bar{\mathcal{D}}_{\dot\alpha}\bar{W}^{\dot\beta})\bar{X}_{\dot\beta}\,,\\ [\Delta,A] &= (\mathcal{D}^a\mathcal{D}_aA)+2(\mathcal{D}^aA)X_a-W^\alpha(\mathcal{D}_\alpha A)-\bar{W}^{\dot\alpha}(\bar{\mathcal{D}}_{\dot\alpha}A)-\\ &\quad-(-1)^{\varepsilon(A)}\,[W^\alpha,A\}\,X_\alpha-(-1)^{\varepsilon(A)}\,[\bar{W}^{\dot\alpha},A\}\,\bar{X}_{\dot\alpha}\,.\end{aligned} \tag{3.15}$$

From these it is clear that summation will generate a series of objects of the form $K_{A_1\ldots A_i}$ (z,t), which we shall refer to as moments of the kernel,[6] as defined in (3.2). Furthermore,

[5]This feature is not particular to the current example. Differential equations of the form (3.13), where the first non-trivial coefficient is absent, arise naturally when applying these techniques to heat kernels associated with "reasonable" operators in superspace with arbitrary dimensions. This is most obvious in ordinary p-dimensional spacetime with Laplace-type operators.

[6]Occasionally we also use this term to collectively refer to all moments and $K(z,t)$ itself.

it is not difficult to show that to order n in this summation, the moments generated have at most $(n+1)$ indices. It is convenient to always place these indices in a specific order: first undotted, then dotted, then spacetime. This can be achieved through the commutation relations (2.20). With such an ordering, the leading term in a moment's asymptotic power series has the following behaviour:[7]

$$K_{A_1\dots A_{p+q}}(z,t) \sim \frac{1}{t^2}\left(\frac{1}{t}\right)^{[p/2]} t^{4-q} = t^{2-q-[p/2]}\,, \qquad q \leq 4\,, \tag{3.16}$$

where $K_{A_1\dots A_{p+q}}(z,t)$ has p spacetime indices, q spinor indices and $[p/2]$ denotes the largest integer part of $p/2$. Moments with greater than two undotted or dotted indices vanish as $X_\alpha X_\beta X_\gamma = \bar{X}_{\dot\alpha}\bar{X}_{\dot\beta}\bar{X}_{\dot\gamma} = 0$.

From these considerations, and by comparison with eq. (3.14), the summation in eq. (3.13) truncates at $n = 2k-5$ when evaluating a_k for $k \geq 5$. Moreover, it turns out that after tracing over gauge indices, the last term in this truncated summation always vanishes due to the cyclic property of the trace, making it necessary to sum only to $n = 2k-6$. In particular, this means that to evaluate a_5 and a_6 one is permitted to truncate at $n=4$ and 6, respectively. More explicitly, the terms with $n = 2k-5$ are always of the form

$$t^{2k-5} M^{\alpha\beta\dot\alpha\dot\beta b_1\dots b_{2k-8}} K_{\alpha\beta\dot\alpha\dot\beta b_1\dots b_{2k-8}}(z,t)\,, \qquad k \geq 5\,, \tag{3.17}$$

where the coefficient M is some graded commutator. The moment in this expression is only ever required to leading order in t, and at this order, it is proportional to the identity matrix in its group indices. Consequently all contributing terms are only proportional to graded commutators, which vanish under the trace.

4. The F^5 terms

In this section, the calculation of the DeWitt-Seeley coefficient a_5 in $\mathcal{N}=4$ supersymmetric Yang-Mills theory is discussed. This determines the F^5 terms in the low-energy effective action.

4.1 Evaluating a_5

Before proceeding, it is instructive to examine the differential eq. (3.13) in a little more detail. Since

$$ad_A^n(BC) = \sum_{m=0}^{n} \frac{n!}{m!(n-m)!} ad_A^{n-m}(B)\, ad_A^m(C)\,, \tag{4.1}$$

if A has even Grassmann parity, then

$$\begin{aligned} ad_\Delta^n(J^a - X^a) &= \frac{\mathrm{i}}{2}(\sigma^a)_{\alpha\dot\alpha} ad_\Delta^n((\bar\theta-\bar\theta')^{\dot\alpha} W^\alpha - (\theta-\theta')^\alpha \bar{W}^{\dot\alpha}) \\ &= \frac{\mathrm{i}}{2}(\sigma^a)_{\alpha\dot\alpha}\Big(\left(\bar\theta-\bar\theta'\right)^{\dot\alpha} ad_\Delta^n(W^\alpha) - (\theta-\theta')^\alpha ad_\Delta^n\left(\bar{W}^{\dot\alpha}\right)\Big) + M_n\,, \end{aligned} \tag{4.2}$$

[7]With arbitrary ordering the behaviour is only slightly more complicated.

where

$$M_n = \frac{\mathrm{i}}{2}(\sigma^a)_{\alpha\dot\alpha}\sum_{m=0}^{n-1}\frac{n!}{m!(n-m)!}\left(ad_\Delta^{n-m-1}\left(\bar W^{\dot\alpha}\right)ad_\Delta^m(W^\alpha)+ad_\Delta^{n-m-1}(W^\alpha)ad_\Delta^m\left(\bar W^{\dot\alpha}\right)\right).$$

The differential eq. (3.13) can then be expressed in the more useful form

$$\begin{aligned}\frac{dK(z,t)}{dt}-\frac{2}{t}K(z,t) &= -\int \mathrm{d}\eta\, X_a\sum_{n=1}^{\infty}\frac{t^n}{(n+1)!}ad_\Delta^n(X^a)e^{t\Delta}-\\ &\quad-\int \mathrm{d}\eta\, X_\alpha\sum_{n=1}^{\infty}\frac{t^n}{(n+1)!}ad_\Delta^n(W^\alpha)e^{t\Delta}-\\ &\quad-\int \mathrm{d}\eta\, \bar X_{\dot\alpha}\sum_{n=1}^{\infty}\frac{t^n}{(n+1)!}ad_\Delta^n(\bar W^{\dot\alpha})e^{t\Delta}-\\ &\quad-\int \mathrm{d}\eta\, X_a\sum_{n=1}^{\infty}\frac{t^n}{(n+1)!}M_n\, e^{t\Delta}\,. \end{aligned} \tag{4.3}$$

To order n in the summation, the last of the four terms on the right hand side will generate moments with at most n indices (whereas the first three generate moments with at most $n+1$). By investigating its powers series behaviour, one ultimately finds that this last term will not contribute when computing a_5, and can safely be ignored.

To illustrate the manner in which the first three terms are evaluated, consider the $n=1$ contribution to the second term. Use of the commutation relations (3.15) gives:

$$\begin{aligned}-\frac{t}{2!}\int \mathrm{d}\eta X_\alpha[\Delta,W^\alpha]e^{t\Delta} &= -\frac{t}{2!}\Big((\mathcal{D}_\alpha A^\alpha)K(z,t)+((\mathcal{D}_\beta C^{\beta\alpha})-A^\alpha)K_\alpha(z,t)+\\ &\quad+(\mathcal{D}_\alpha E^{\alpha\dot\alpha})K_{\dot\alpha}(z,t)+(\mathcal{D}_\alpha B^{\alpha a})K_a(z,t)-\\ &\quad-B^{\alpha a}K_{\alpha a}(z,t)+C^{\alpha\beta}K_{\alpha\beta}(z,t)+E^{\alpha\dot\alpha}K_{\alpha\dot\alpha}(z,t)\Big) \end{aligned} \tag{4.4}$$

with

$$\begin{aligned} A^\alpha &= (\mathcal{D}^a\mathcal{D}_a W^\alpha)-W^\beta(\mathcal{D}_\beta W^\alpha)\,, \qquad B^{\alpha a}=2(\mathcal{D}^a W^\alpha)\,,\\ C^{\alpha\beta} &= \{W^\alpha,W^\beta\}, \qquad E^{\alpha\dot\alpha}=\{W^\alpha,\bar W^{\dot\alpha}\}\,. \end{aligned} \tag{4.5}$$

This is further simplified by the vanishing of the coefficient of $K_{\dot\alpha}(z,t)$ due to chirality and the equations of motion, and $C^{\alpha\beta}K_{\alpha\beta}(z,t)$ vanishes due to the symmetry/antisymmetry of its indices. Furthermore, the terms involving $t\,K(z,t)$ and $t\,K_a(z,t)$ will not contribute to the order of interest, since after expansion both have will have leading terms of order t^3. Curiously, and apparently contrary to eq. (3.14), one also finds a term, $t\,K_{\alpha\dot\alpha}(z,t)$, of leading order t, but all such terms are found to cancel (as they must) when considering the complete right hand side of (4.3).

There now remains the problem of expanding the contributing moments to the required order in t. In the current example this does not pose any additional difficulties since it is only necessary to expand all surviving moments to leading order, and as explained above, such expressions are readily obtained by directly expanding the exponential. For example:

$$K_\alpha(z,t)=\int \mathrm{d}\eta\, X_\alpha e^{t\Delta}=-\frac{1}{16\pi^2}\frac{4t}{3!}\left(W_\alpha\bar W^2+\bar W^2W_\alpha-\bar W_{\dot\alpha}W_\alpha\bar W^{\dot\alpha}\right)+\mathcal{O}(t^2)\,, \tag{4.6}$$

where the k integral has also been performed (after Wick rotation to a euclidean metric). In essence the problem of computing the kernel to subleading order has been reduced to computing several moments to leading order.

Carrying out this procedure to order t^2 for the entire right hand side of eq. (4.3) for $n = 1$ to 4, and using the cyclicity of the trace, $\mathrm{Tr}(a_5)$ can be identified:

$$\begin{aligned}
\mathrm{Tr}(a_5) = \frac{1}{90}\,\mathrm{Tr}\Big(&10\Big((\mathcal{D}^a\mathcal{D}_aW^\alpha)W_\alpha\bar{W}^2 + (\mathcal{D}^a\mathcal{D}_aW^\alpha)\bar{W}^2W_\alpha - (\mathcal{D}^a\mathcal{D}_aW^\alpha)\bar{W}_{\dot\alpha}W_\alpha\bar{W}^{\dot\alpha}\Big) + \\
&+ 11\Big((\mathcal{D}^aW^\alpha)(\mathcal{D}_aW_\alpha)\bar{W}^2 + (\mathcal{D}^aW^\alpha)(\mathcal{D}_a\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}W_\alpha - \\
&\quad - (\mathcal{D}^aW^\alpha)(\mathcal{D}_a\bar{W}_{\dot\alpha})W_\alpha\bar{W}^{\dot\alpha}\Big) + \\
&+ 4\Big((\mathcal{D}^aW^\alpha)W_\alpha(\mathcal{D}_a\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha} + (\mathcal{D}^aW^\alpha)\bar{W}_{\dot\alpha}(\mathcal{D}_a\bar{W}^{\dot\alpha})W_\alpha - \\
&\quad - (\mathcal{D}^aW^\alpha)\bar{W}_{\dot\alpha}(\mathcal{D}_aW_\alpha)\bar{W}^{\dot\alpha}\Big) + 3(\mathcal{D}_\alpha W^\beta)W^\alpha W_\beta\bar{W}^2 + \\
&+ 9(\mathcal{D}_\alpha W^\beta)W^\alpha\bar{W}^2W_\beta + 3(\mathcal{D}_\alpha W^\beta)\bar{W}^2W^\alpha W_\beta + \\
&+ 3(\mathcal{D}_\alpha W^\beta)\bar{W}_{\dot\alpha}W^\alpha W_\beta\bar{W}^{\dot\alpha} - 6(\mathcal{D}_\alpha W^\beta)W^\alpha\bar{W}_{\dot\alpha}W_\beta\bar{W}^{\dot\alpha} - \\
&- 6(\mathcal{D}_\alpha W^\beta)\bar{W}_{\dot\alpha}W^\alpha\bar{W}^{\dot\alpha}W_\beta\Big) + \text{c.c.}\,. \qquad (4.7)
\end{aligned}$$

Here the complex conjugate of any term is effectively obtained by replacing all undotted spinor indices (and unbarred objects) by dotted spinor indices (and barred objects) and vice-versa. Integrating by parts, the result can be brought into the more compact form

$$\begin{aligned}
\mathrm{Tr}(a_5) = \frac{1}{30}\,\mathrm{Tr}\Big(&2\Big((\mathcal{D}^a\mathcal{D}_aW^\alpha)W_\alpha\bar{W}^2 + (\mathcal{D}^a\mathcal{D}_aW^\alpha)\bar{W}^2W_\alpha - (\mathcal{D}^a\mathcal{D}_aW^\alpha)\bar{W}_{\dot\alpha}W_\alpha\bar{W}^{\dot\alpha}\Big) + \\
&+ \Big((\mathcal{D}^aW^\alpha)(\mathcal{D}_aW_\alpha)\bar{W}^2 + (\mathcal{D}^aW^\alpha)(\mathcal{D}_a\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}W_\alpha - \\
&\quad - (\mathcal{D}^aW^\alpha)(\mathcal{D}_a\bar{W}_{\dot\alpha})W_\alpha\bar{W}^{\dot\alpha}\Big) + \\
&+ 5\Big((\mathcal{D}_\alpha W^\beta)W^\alpha W_\beta\bar{W}^2 - (\mathcal{D}_\alpha W^\beta)W^\alpha\bar{W}_{\dot\alpha}W_\beta\bar{W}^{\dot\alpha}\Big)\Big) + \text{c.c.}\,. \qquad (4.8)
\end{aligned}$$

The corresponding piece of the one-loop effective action can immediately be deduced by insertion into eq. (2.23).

4.2 a_5 at the component level

We are now in a position to extract the component form of $\mathrm{Tr}(a_5)$. We consider only the contribution containing the field strength F_{ab} and its covariant derivatives. It is natural to split the result into two parts and compute their component fields separately. Firstly, consider only terms with two covariant derivatives:

$$\begin{aligned}
\frac{1}{30}\,\mathrm{Tr}\Big(&2\Big((\mathcal{D}^a\mathcal{D}_aW^\alpha)W_\alpha\bar{W}^2 + (\mathcal{D}^a\mathcal{D}_aW^\alpha)\bar{W}^2W_\alpha - (\mathcal{D}^a\mathcal{D}_aW^\alpha)\bar{W}_{\dot\alpha}W_\alpha\bar{W}^{\dot\alpha}\Big) + \\
&+ \Big((\mathcal{D}^aW^\alpha)(\mathcal{D}_aW_\alpha)\bar{W}^2 + (\mathcal{D}^aW^\alpha)(\mathcal{D}_a\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}W_\alpha - \\
&\quad - (\mathcal{D}^aW^\alpha)(\mathcal{D}_a\bar{W}_{\dot\alpha})W_\alpha\bar{W}^{\dot\alpha}\Big)\Big) + \text{c.c.}\,.
\end{aligned}$$

Using standard techniques (for example, see [35]), it is not difficult to show that the relevant component part of this superfield expression is:

$$\frac{1}{30}\,\mathrm{Tr}\,\Big(2\Big((\nabla^e F^{ab})(\nabla_e F_{bc})F^{cd}F_{da} + (\nabla^e F^{ab})(\nabla_e F^{cd})F_{bc}F_{da} + (\nabla^e F^{ab})(\nabla_e F_{ca})F_{bd}F^{dc}\Big) -$$
$$- \frac{1}{2}\Big((\nabla^e F^{ab})(\nabla_e F_{ab})F^{cd}F_{cd} + (\nabla^e F^{ab})(\nabla_e F^{cd})F_{ab}F_{cd} + (\nabla^e F^{ab})(\nabla_e F^{cd})F_{cd}F_{ab}\Big) +$$
$$+ 4\Big((\nabla^2 F^{ab})F_{bc}F^{cd}F_{da} + (\nabla^2 F^{ab})F^{cd}F_{bc}F_{da} + (\nabla^2 F^{ab})F_{ca}F_{bd}F^{dc}\Big) -$$
$$- \Big((\nabla^2 F^{ab})F_{ab}F^{cd}F_{cd} + (\nabla^2 F^{ab})F^{cd}F_{ab}F_{cd} + (\nabla^2 F^{ab})F^{cd}F_{cd}F_{ab}\Big)\Big)\,, \tag{4.9}$$

where

$$\begin{aligned} \nabla_a &= \partial_a - \mathrm{i}A_a\,, \qquad [\nabla_a, \nabla_b] = -\mathrm{i}F_{ab}\,, \\ F_{ab} &= \partial_a A_b - \partial_b A_a - \mathrm{i}[A_a, A_b]\,, \qquad \nabla_c(F_{ab}) = \partial_c F_{ab} - \mathrm{i}[A_c, F_{ab}]\,. \end{aligned} \tag{4.10}$$

The $\nabla^2 F$ terms can be converted into F^2-type terms using

$$\nabla^2 F_{ab} = 2\mathrm{i}\,(F_{ac}F^c{}_b - F_{bc}F^c{}_a)\,, \tag{4.11}$$

thus generating some F^5 terms. The resulting expression can be further simplified by recognizing that out of the six distinct F^5 structures, only four are linearly independent, as seen via the following identities:

$$\begin{aligned} \mathrm{Tr}(F^{ab}F^{de}F_{bc}F^c{}_aF_{de}) = {}& \frac{1}{5}\,\mathrm{Tr}(F^{ab}F_{bc}F^{cd}F_{de}F^e{}_a) - \mathrm{Tr}(F^{ab}F_{bc}F^{cd}F^e{}_aF_{de}) + \\ &+ \mathrm{Tr}(F^{ab}F^{cd}F_{bc}F^e{}_aF_{de}) + \frac{3}{5}\,\mathrm{Tr}(F^{ab}F^{cd}F^e{}_aF_{bc}F_{de}) \end{aligned} \tag{4.12}$$

and

$$\begin{aligned} \mathrm{Tr}(F^{ab}F_{bc}F^c{}_aF^{de}F_{de}) = {}& -\frac{3}{5}\,\mathrm{Tr}(F^{ab}F_{bc}F^{cd}F_{de}F^e{}_a) + \mathrm{Tr}(F^{ab}F_{bc}F^{cd}F^e{}_aF_{de}) + \\ &+ \mathrm{Tr}(F^{ab}F^{cd}F_{bc}F^e{}_aF_{de}) + \frac{1}{5}\,\mathrm{Tr}(F^{ab}F^{cd}F^e{}_aF_{bc}F_{de})\,. \end{aligned} \tag{4.13}$$

Using these, eq. (4.9) reduces to

$$\frac{1}{30}\,\mathrm{Tr}\,\Big(2\Big((\nabla^e F^{ab})(\nabla_e F_{bc})F^{cd}F_{da} + (\nabla^e F^{ab})(\nabla_e F^{cd})F_{bc}F_{da} + (\nabla^e F^{ab})(\nabla_e F_{ca})F_{bd}F^{dc}\Big) -$$
$$- \frac{1}{2}\Big((\nabla^e F^{ab})(\nabla_e F_{ab})F^{cd}F_{cd} + (\nabla^e F^{ab})(\nabla_e F^{cd})F_{ab}F_{cd} +$$
$$+ (\nabla^e F^{ab})(\nabla_e F^{cd})F_{cd}F_{ab}\Big) +$$
$$+ 4\mathrm{i}\Big(F^{ab}F_{bc}F^{cd}F_{de}F^e{}_a + 3F^{ab}F_{bc}F^{cd}F^e{}_aF_{de} -$$
$$- F^{ab}F^{cd}F_{bc}F^e{}_aF_{de} + F^{ab}F^{cd}F^e{}_aF_{bc}F_{de}\Big)\Big)\,. \tag{4.14}$$

The bosonic component of $\mathrm{Tr}(a_5)$ coming from the terms in (4.8) with a single covariant derivative is

$$-\frac{\mathrm{i}}{12}\,\mathrm{Tr}\,\Big(2F^{ab}F_{bc}F^{cd}F_{de}F^{e}{}_{a} + 4F^{ab}F_{bc}F^{cd}F^{e}{}_{a}F_{de} - 2F^{ab}F^{cd}F_{bc}F^{e}{}_{a}F_{de} + \\ + F^{ab}F_{bc}F^{c}{}_{a}F^{de}F_{de} + F^{ab}F^{de}F_{bc}F^{c}{}_{a}F_{de}\Big)\,, \tag{4.15}$$

which, after the application of (4.12) and (4.13) becomes

$$-\frac{\mathrm{i}}{15}\,\mathrm{Tr}\,\Big(2F^{ab}F_{bc}F^{cd}F_{de}F^{e}{}_{a} + 5F^{ab}F_{bc}F^{cd}F^{e}{}_{a}F_{de} + F^{ab}F^{cd}F^{e}{}_{a}F_{bc}F_{de}\Big)\,. \tag{4.16}$$

Finally, adding (4.14) and (4.16), one obtains the overall bosonic component of $\mathrm{Tr}(a_5)$,

$$\frac{1}{30}\,\mathrm{Tr}\,\Big(2\Big((\nabla^e F^{ab})(\nabla_e F_{bc})F^{cd}F_{da} + (\nabla^e F^{ab})(\nabla_e F^{cd})F_{bc}F_{da} + (\nabla^e F^{ab})(\nabla_e F_{ca})F_{bd}F^{dc}\Big) - \\ -\frac{1}{2}\Big((\nabla^e F^{ab})(\nabla_e F_{ab})F^{cd}F_{cd} + (\nabla^e F^{ab})(\nabla_e F^{cd})F_{ab}F_{cd} + (\nabla^e F^{ab})(\nabla_e F^{cd})F_{cd}F_{ab}\Big) + \\ + 2\mathrm{i}\Big(F^{ab}F_{bc}F^{cd}F^{e}{}_{a}F_{de} - 2F^{ab}F^{cd}F_{bc}F^{e}{}_{a}F_{de} + F^{ab}F^{cd}F^{e}{}_{a}F_{bc}F_{de}\Big)\Big)\,. \tag{4.17}$$

After conversion to the basis used in [13, 14] (see appendix A), we find exact agreement, up to an overall multiplicative constant, with eq. (1.3), which describes the $(\alpha')^3$ terms in the non-abelian Born-Infeld action.

Conversely we do not find agreement with the results of [12], eq. (1.2), where the component form of the $\mathcal{N}=4$ super Yang-Mills one-loop effective action is extracted in several pieces through supergraph techniques.[8]

5. The F^6 terms

5.1 Expanding moments

Employing the procedure outlined above to compute a_k for $k > 5$ will necessarily involve asymptotically expanding moments to higher than leading order. A prescription will therefore be required if this scheme is to be generalized. It is possible to appeal to a set of techniques similar to those already seen in the previous sections.

More specifically, to evaluate any moment to arbitrary order, one proceeds iteratively by using the following generalizations of the identities (3.3), (3.9) and (3.10):

$$0 = \int \mathrm{d}\eta \frac{\partial}{\partial k_b}\left(X_{A_1}\ldots X_{A_n}e^{t\Delta}\right), \tag{5.1}$$

$$0 = \int \mathrm{d}\eta \frac{\partial}{\partial \epsilon_\beta}\left(X_{A_1}\ldots X_{A_n}e^{t\Delta}\right), \tag{5.2}$$

$$0 = \int \mathrm{d}\eta \frac{\partial}{\partial \bar{\epsilon}_{\dot\beta}}\left(X_{A_1}\ldots X_{A_n}e^{t\Delta}\right); \tag{5.3}$$

[8] It is perhaps worth pointing out that separate agreement (up to overall multiplicative constants) is found between our eqs. (4.9) and (4.16), and the corresponding equations in [12], (6.2) and (4.13), respectively. However, the multiplicative factors differ in each case, and so on assembling the final result, a discrepancy emerges.

or by differentiation with respect to t as in (3.1):

$$\frac{d^m K_{A_1\ldots A_n}(z,t)}{dt^m} = \int \mathrm{d}\eta X_{A_1}\ldots X_{A_n}\Delta^m e^{t\Delta}\,; \tag{5.4}$$

or by using a combination of the two as in (3.13). Of course, none of this actually computes the moment directly, but is used with the intention of expressing it in terms of other moments with the same number or more indices, which are generally easier to compute directly. Using this procedure, expanding a moment to some order will usually require knowledge of the expansion of several other moments to the same or lower order. Consequently at some point it will be necessary to evaluate at least one moment directly by expanding the exponential.

5.2 The moment hierarchy and a_6

Computing a_6 involves summing from $n = 1$ to 6 on the right hand side of (4.3), which generates a hierarchy of moments, a partial list being given below (all but $K(z,t)$ required to subleading order):

$$\begin{array}{c}
K_{\alpha\beta\dot\alpha\dot\beta ab}(z,t) \\
K_{\alpha\beta\dot\alpha\dot\beta a}(z,t) \quad K_{\alpha\beta\dot\alpha ab}(z,t) \quad K_{\alpha\dot\alpha\dot\beta ab}(z,t) \quad K_{\alpha\beta\dot\alpha\dot\beta}(z,t) \\
K_{\alpha\beta\dot\alpha a}(z,t) \quad K_{\alpha\dot\alpha\dot\beta a}(z,t) \quad K_{\alpha\beta ab}(z,t) \quad K_{\alpha\dot\alpha ab}(z,t) \quad K_{\dot\alpha\dot\beta ab}(z,t) \quad K_{\alpha\beta\dot\alpha}(z,t) \quad K_{\alpha\dot\alpha\dot\beta}(z,t) \\
K_{\alpha\beta a}(z,t) \quad K_{\alpha\dot\alpha a}(z,t) \quad K_{\dot\alpha\dot\beta a}(z,t) \quad K_{\alpha ab}(z,t) \quad K_{\dot\alpha ab}(z,t) \quad K_{\alpha\beta}(z,t) \quad K_{\alpha\dot\alpha}(z,t) \quad K_{\dot\alpha\dot\beta}(z,t) \\
\vdots \\
K_{ab}(z,t) \quad K_{\alpha a}(z,t) \quad K_{\dot\alpha a}(z,t) \quad K_{\alpha}(z,t) \quad K_{\dot\alpha}(z,t) \\
K(z,t)
\end{array}$$

Generally speaking, the following structure is present: from top to bottom the moments decrease in the number of indices, increase in difficulty of expansion, and the exponent of t in the leading order term increases (each row contains moments with the same leading order). From left to right, the moments decrease in their difficulty of expansion, and clearly many are related by complex conjugation.

In the prescription outlined above, the expansion of any moment hinges on having computed the expansion of a number of those next to or above it in the hierarchy, so naturally one begins at the top and works down. To be more explicit, consider the following three examples which cover all important points.

$K_{\alpha\beta\dot\alpha\dot\beta}(z,t)$ turns out to be a rather important object in this hierarchy, in that all others can be expressed in terms of it. It's power series to subleading order is not difficult to compute by directly expanding the exponential, and takes the simple form:

$$K_{\alpha\beta\dot\alpha\dot\beta}(z,t) = -\frac{1}{16\pi^2}\frac{4}{t^2}\varepsilon_{\alpha\beta}\varepsilon_{\dot\alpha\dot\beta} + \mathcal{O}(t^0)\,, \tag{5.5}$$

where the t^{-1} term vanishes due to the equations of motion.

Computing $K_{\alpha\beta\dot{\alpha}}(z,t)$ involves the identity

$$0 = \int \mathrm{d}\eta \frac{\partial}{\partial \bar{\epsilon}_{\dot{\gamma}}} (X_\alpha X_\beta \bar{X}_{\dot{\alpha}} \bar{X}_{\dot{\beta}} e^{t\Delta}) \tag{5.6}$$

which, after the contraction of $\dot{\beta}$ and $\dot{\gamma}$, leads to

$$K_{\alpha\beta\dot{\alpha}}(z,t) = \int \mathrm{d}\eta\; X_\alpha X_\beta \bar{X}_{\dot{\alpha}} \bar{X}_{\dot{\beta}} \sum_{n=0}^{\infty} \frac{t^{n+1}}{(n+1)!} ad_\Delta^n (\bar{W}^{\dot{\beta}}) e^{t\Delta} \,. \tag{5.7}$$

To leading or subleading order, the summation can be truncated at $n = 0$ or 2, respectively. Alternatively, one may have chosen to start with the identity

$$0 = \int \mathrm{d}\eta \frac{\partial}{\partial k_b} \left(X_\alpha X_\beta \bar{X}_{\dot{\alpha}} X_a \, e^{t\Delta} \right) \tag{5.8}$$

to obtain an expression for $K_{\alpha\beta\dot{\alpha}}(z,t)$, but this ends up being far more complicated. In general, if the moment in question has less than four spinor indices, it is more convenient to chose the identities (5.2) or (5.3) rather than (5.1). However, if there are four spinor indices there is no choice and (5.1) must be used.

Summing from $n = 0$ to 2 in (5.7), one finds that to subleading order, $K_{\alpha\beta\dot{\alpha}}(z,t)$ can be expressed in terms of

$$t^3 K_{\alpha\beta\dot{\alpha}\dot{\beta}ab}(z,t)\,, \qquad t^2 K_{\alpha\beta\dot{\alpha}\dot{\beta}}(z,t)\,, \qquad t K_{\alpha\beta\dot{\alpha}\dot{\beta}}(z,t)\,, \qquad \text{and} \qquad t K_{\alpha\beta\dot{\alpha}}(z,t)\,,$$

where only $K_{\alpha\beta\dot{\alpha}\dot{\beta}}(z,t)$ is actually required to subleading order. Notice that $K_{\alpha\beta\dot{\alpha}}(z,t)$ is actually expressed in terms of itself (multiplied by t). This is a typical feature of this approach, and one can either rely on the fact that $K_{\alpha\beta\dot{\alpha}}(z,t)$ is already known to leading order, or bring it to the left hand side and premultiply both sides by an inverse operator (to appropriate order) to generate an new expression for $K_{\alpha\beta\dot{\alpha}}(z,t)$ in terms of only $K_{\alpha\beta\dot{\alpha}\dot{\beta}ab}(z,t)$ and $K_{\alpha\beta\dot{\alpha}\dot{\beta}}(z,t)$.

As a final example, consider expanding the moment $K_{\alpha\beta}(z,t)$ to subleading order. In this case it is far more convenient to differentiate with respect to t. The power series expansion of $K_{\alpha\beta}(z,t)$ will look like

$$K_{\alpha\beta}(z,t) = A + t\,B + \mathcal{O}\left(t^2\right), \tag{5.9}$$

and so,

$$\frac{dK_{\alpha\beta}(z,t)}{dt} = B + \mathcal{O}(t)\,. \tag{5.10}$$

Therefore, after a little work, to order unity in t (i.e. t^0)

$$\frac{dK_{\alpha\beta}(z,t)}{dt} = K_{\alpha\beta a}{}^{a}(z,t) - \bar{W}^{\dot{\alpha}} K_{\alpha\beta\dot{\alpha}}(z,t)\,. \tag{5.11}$$

So if both $K_{\alpha\beta ab}(z,t)$ and $K_{\alpha\beta\dot{\alpha}}(z,t)$, which are higher up the hierarchy, are known to subleading order (to order unity in t), $K_{\alpha\beta}(z,t)$ can immediately be evaluated to subleading order (i.e. identify B). Additionally this generates the leading order identity

$$K_{\alpha\beta a}{}^{a}(z,t) = \bar{W}^{\dot{\alpha}} K_{\alpha\beta\dot{\alpha}}(z,t)\,, \tag{5.12}$$

which serves as a useful consistency check.

Having summed the right hand side of (4.3) from $n = 1$ to 6 and expanded all surviving terms to order t^3, a_6 can be identified (and of course a_5 is also recovered in this process). The final result is given in appendix B. Due to its size, and the fact that there are many equivalent ways of presenting the result, it is a significant challenge to find the most compact and symmetric looking expression. By extensive use of commutation relations, equations of motion and the cyclicity of the trace, the result is brought into a manifestly real form involving only seven distinct types of terms, each listed schematically below (where G_{ab} was defined in (2.20)):

$$W^2\,\bar{W}^2\,\mathcal{D}_a^4\,,\qquad W^2\,\bar{W}^2\,G_{ab}\,\mathcal{D}_a^2\,,\qquad W^3\,\bar{W}^2\,\mathcal{D}_\alpha\,\mathcal{D}_a^2\,,\qquad W^2\,\bar{W}^3\,\bar{\mathcal{D}}_{\dot\alpha}\,\mathcal{D}_a^2\,,$$
$$W^3\,\bar{W}^3\,\mathcal{D}_\alpha\,\bar{\mathcal{D}}_{\dot\alpha}\,,\qquad W^4\,\bar{W}^2\,\mathcal{D}_\alpha^2\,,\qquad W^2\,\bar{W}^4\,\bar{\mathcal{D}}_{\dot\alpha}^2\,.$$

Here, for example, $W^2\,\bar{W}^2\,\mathcal{D}_a^4$ is taken to mean terms which contain (some specific permutation and contraction of) two chiral superfield strengths, two antichiral superfield strengths and four spacetime covariant derivatives.

Again the corresponding contribution to the effective action can be obtained by inspection, but integrating by parts offers little in the way of simplification. Extraction of the component form of a_6 is now in principle straightforward, and contains F^6-type field strength terms.

6. Discussion

In this paper, the F^5 and F^6 terms in the one-loop low-energy effective action for $\mathcal{N} = 4$ supersymmetric Yang-Mills theory have been computed in $\mathcal{N} = 1$ superfield form. As noted in the introduction, the F^5 terms are consistent with the result (1.3) obtained from superstring theory, providing evidence for a unique form for the non-abelian Born-Infeld action at this order. The F^6 terms in the low-energy effective action have not previously been computed, and comparison with recent superstring results [30, 31, 32] remains to be carried out.

It is possible to perform a non-trivial test on the F^6 results. The form of the one-loop low-energy effective action is known in the abelian case in the constant field strength approximation [40, 41], and the coefficient of F^6 is zero. Inspection of a_6 reveals that in the abelian limit, F^6 contributions for constant field strength can come only from terms of the form: $W^4\bar{W}^2\mathcal{D}_\alpha^2$ and $W^2\bar{W}^4\bar{\mathcal{D}}_{\dot\alpha}^2$ (which encompass the last terms in a_6 as it is given). Explicitly, the result reduces to

$$\begin{aligned}
\frac{1}{2520}\Big(& (\mathcal{D}_\alpha W^\beta)(\mathcal{D}_\beta W^\alpha)W^2\bar{W}^2(5+5+5+33+19+19)\,+ \\
&+\,(\mathcal{D}_\alpha W^\beta)(\mathcal{D}_\beta W^\alpha)W^2\bar{W}^2(-6+8+8-6-6+22)\,+ \\
&+\,(\mathcal{D}_\alpha W^\beta)(\mathcal{D}_\beta W^\alpha)W^2\bar{W}^2\left(9-\frac{5}{2}-\frac{5}{2}-5\right)+ \\
&+\,(\mathcal{D}_\alpha W^\beta)(\mathcal{D}_\gamma W^\alpha)W_\beta W^\gamma\bar{W}^2(14+14+14+42+28+28)\,+ \\
&+\,14(\mathcal{D}_\alpha W^\beta)(\mathcal{D}_\gamma W^\alpha)W_\beta W^\gamma\bar{W}^2(1+1+2)\,+
\end{aligned}$$

$$+\,14(\mathcal{D}_\alpha W^\beta)(\mathcal{D}_\gamma W^\alpha)W_\beta W^\gamma \bar{W}^2\Big) + \text{c.c.} =$$

$$= \frac{1}{24}\Big((\mathcal{D}_\alpha W^\beta)(\mathcal{D}_\beta W^\alpha)W^2\bar{W}^2 + 2(\mathcal{D}_\alpha W^\beta)(\mathcal{D}_\gamma W^\alpha)W_\beta W^\gamma \bar{W}^2\Big) + \text{c.c.}\,.$$

The second term can be rearranged into the same form as the first, since in the abelian case,

$$W_\alpha W_\beta = \frac{1}{2}\varepsilon_{\alpha\beta}W^2 \Longrightarrow$$

$$\Longrightarrow (\mathcal{D}_\alpha W^\beta)(\mathcal{D}_\gamma W^\alpha)W_\beta W^\gamma \bar{W}^2 = -\frac{1}{2}(\mathcal{D}_\alpha W^\beta)(\mathcal{D}_\beta W^\alpha)W^2\bar{W}^2\,,$$

and indeed one finds non-trivial cancellation, consistent with [40, 41].

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A. Change of basis

In this appendix we briefly outline the transformation from the basis used in our result (4.17), to that of (1.3). For simplicity we introduce the following notation:

$$\begin{aligned}
s_{0,0} &= \mathrm{Tr}\left(F^{ab}F_{bc}F^{cd}F_{de}F^{e}{}_{a}\right), & s_{0,1} &= \mathrm{Tr}\left(F^{ab}F_{bc}F^{cd}F^{e}{}_{a}F_{de}\right),\\
s_{0,2} &= \mathrm{Tr}\left(F^{ab}F^{cd}F_{bc}F^{e}{}_{a}F_{de}\right), & s_{0,3} &= \mathrm{Tr}\left(F^{ab}F^{cd}F^{e}{}_{a}F_{bc}F_{de}\right),\\
s_{0,4} &= \mathrm{Tr}\left(F^{ab}F_{bc}F^{c}{}_{a}F^{de}F_{de}\right), & s_{0,5} &= \mathrm{Tr}\left(F^{ab}F^{de}F_{bc}F^{c}{}_{a}F_{de}\right),\\
s_{1,0} &= \mathrm{Tr}\left((\nabla^e F^{ab})(\nabla_e F_{ab})F^{cd}F_{cd}\right), & s_{1,1} &= \mathrm{Tr}\left((\nabla^e F^{ab})(\nabla_e F^{cd})F_{ab}F_{cd}\right),\\
s_{1,2} &= \mathrm{Tr}\left((\nabla^e F^{ab})(\nabla_e F^{cd})F_{cd}F_{ab}\right), & s_{1,3} &= \mathrm{Tr}\left((\nabla^e F^{ab})(\nabla_e F_{bc})F^{cd}F_{da}\right),\\
s_{1,4} &= \mathrm{Tr}\left((\nabla^e F^{ab})(\nabla_e F_{ca})F_{bd}F^{dc}\right), & s_{1,5} &= \mathrm{Tr}\left((\nabla^e F^{ab})(\nabla_e F^{cd})F_{bc}F_{da}\right),\\
s_{1,6} &= \mathrm{Tr}\left((\nabla^e F^{ab})F_{da}(\nabla_e F_{bc})F^{cd}\right), & s_{1,7} &= \mathrm{Tr}\left((\nabla^e F^{ab})F^{cd}(\nabla_e F_{ab})F_{cd}\right),\\
s_{2,0} &= \mathrm{Tr}\left((\nabla^e F^{ab})F^{cd}(\nabla_a F_{bc})F_{de}\right), & s_{2,1} &= \mathrm{Tr}\left(F^{ab}(\nabla_a F^{cd})(\nabla^e F_{bc})F_{de}\right),\\
s_{2,2} &= \mathrm{Tr}\left(F^{ab}(\nabla_a F^{bc})F^{de}(\nabla_e F_{cd})\right).
\end{aligned}$$

Using the equations of motion, the Bianchi identity, integration by parts and the cyclic property of the trace, one can establish the following identities:

$$\begin{aligned}
s_{0,4} &= -\frac{3}{5}s_{0,0} + s_{0,1} + s_{0,2} + \frac{1}{5}s_{0,3}\,, \qquad s_{0,5} = \frac{1}{5}s_{0,0} - s_{0,1} + s_{0,2} + \frac{3}{5}s_{0,3}\,,\\
s_{1,1} &= -\frac{1}{2}s_{1,7} + 2\mathrm{i}s_{0,5}\,, \qquad s_{1,2} = -s_{1,0} - 4s_{2,2} + 4\mathrm{i}s_{0,4}\,,\\
s_{1,3} &= -\mathrm{i}s_{0,0} - \mathrm{i}s_{0,1} + \mathrm{i}s_{0,2} + 3\mathrm{i}s_{0,3} - \frac{\mathrm{i}}{2}s_{0,4} - \mathrm{i}s_{0,5} - 2s_{1,4} - s_{1,6} +\\
&\quad + \frac{3}{8}s_{1,7} - 4s_{2,0} + 4s_{2,1} - s_{2,2}\,,\\
s_{1,5} &= -s_{1,6} - s_{1,4} - 2\mathrm{i}s_{0,1} + 2\mathrm{i}s_{0,2}\,.
\end{aligned}$$

In this notation, the bosonic component of $\mathrm{Tr}(a_5)$, eq. (4.17), takes the form:

$$\frac{1}{30}\,\mathrm{Tr}\left(2(s_{1,3}+s_{1,4}+s_{1,5})-\frac{1}{2}(s_{1,0}+s_{1,1}+s_{1,2})+2\mathrm{i}(s_{0,1}-2s_{0,2}+s_{0,3})\right). \tag{A.1}$$

Using the above relations, elimination of $s_{1,1}$, $s_{1,2}$, $s_{1,3}$ and $s_{1,5}$, followed by the further elimination of $s_{0,4}$ and $s_{0,5}$, yields the following expression:

$$\frac{4}{15}\,\mathrm{Tr}\left(s_{2,1}-s_{2,0}-\frac{1}{2}s_{1,4}-\frac{1}{2}s_{1,6}+\frac{1}{8}s_{1,7}-\frac{\mathrm{i}}{10}s_{0,0}-\frac{\mathrm{i}}{2}s_{0,1}-\frac{\mathrm{i}}{2}s_{0,2}+\frac{7\mathrm{i}}{10}s_{0,3}\right). \tag{A.2}$$

which up to an overall multiplicative factor, is eq. (1.3).

B. $\mathrm{Tr}(a_6)$

$$\begin{aligned}
&\mathrm{Tr}(a_6)=\\
&=\frac{1}{2520}\,\mathrm{Tr}\Big(28(\mathcal{D}^a\mathcal{D}_a\mathcal{D}^b\mathcal{D}_bW^\alpha)\Big(W_\alpha\bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}+\bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}W_\alpha-\bar{W}_{\dot\alpha}W_\alpha\bar{W}^{\dot\alpha}\Big)+\\
&\qquad+62(\mathcal{D}^a\mathcal{D}^b\mathcal{D}_bW^\alpha)\Big((\mathcal{D}_aW_\alpha)\bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}+(\mathcal{D}_a\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}W_\alpha-(\mathcal{D}_a\bar{W}_{\dot\alpha})W_\alpha\bar{W}^{\dot\alpha}\Big)+\\
&\qquad+62(\mathcal{D}^a\mathcal{D}^b\mathcal{D}_bW^\alpha)\Big(W_\alpha\bar{W}_{\dot\alpha}(\mathcal{D}_a\bar{W}^{\dot\alpha})+\bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}(\mathcal{D}_aW_\alpha)-\bar{W}_{\dot\alpha}W_\alpha(\mathcal{D}_a\bar{W}^{\dot\alpha})\Big)+\\
&\qquad+44(\mathcal{D}^a\mathcal{D}^b\mathcal{D}_bW^\alpha)\Big(W_\alpha(\mathcal{D}_a\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}+\bar{W}_{\dot\alpha}(\mathcal{D}_a\bar{W}^{\dot\alpha})W_\alpha-\bar{W}_{\dot\alpha}(\mathcal{D}_aW_\alpha)\bar{W}^{\dot\alpha}\Big)+\\
&\qquad+48(\mathcal{D}^b\mathcal{D}_bW^\alpha)\Big((\mathcal{D}^a\mathcal{D}_aW_\alpha)\bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}+(\mathcal{D}^a\mathcal{D}_a\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}W_\alpha-(\mathcal{D}^a\mathcal{D}_a\bar{W}_{\dot\alpha})W_\alpha\bar{W}^{\dot\alpha}\Big)+\\
&\qquad+22(\mathcal{D}^b\mathcal{D}_bW^\alpha)\Big(W_\alpha(\mathcal{D}^a\mathcal{D}_a\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}+\\
&\qquad\qquad+\bar{W}_{\dot\alpha}(\mathcal{D}^a\mathcal{D}_a\bar{W}^{\dot\alpha})W_\alpha-\bar{W}_{\dot\alpha}(\mathcal{D}^a\mathcal{D}_aW_\alpha)\bar{W}^{\dot\alpha}\Big)+\\
&\qquad+50(\mathcal{D}^b\mathcal{D}_bW^\alpha)\Big((\mathcal{D}^aW_\alpha)(\mathcal{D}_a\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}+\\
&\qquad\qquad+(\mathcal{D}^a\bar{W}_{\dot\alpha})(\mathcal{D}_a\bar{W}^{\dot\alpha})W_\alpha-(\mathcal{D}^a\bar{W}_{\dot\alpha})(\mathcal{D}_aW_\alpha)\bar{W}^{\dot\alpha}\Big)+\\
&\qquad+50(\mathcal{D}^b\mathcal{D}_bW^\alpha)\Big(W_\alpha(\mathcal{D}^a\bar{W}_{\dot\alpha})(\mathcal{D}_a\bar{W}^{\dot\alpha})+\\
&\qquad\qquad+\bar{W}_{\dot\alpha}(\mathcal{D}^a\bar{W}^{\dot\alpha})(\mathcal{D}_aW_\alpha)-\bar{W}_{\dot\alpha}(\mathcal{D}^aW_\alpha)(\mathcal{D}_a\bar{W}^{\dot\alpha})\Big)+\\
&\qquad+40(\mathcal{D}^b\mathcal{D}_bW^\alpha)\Big((\mathcal{D}^aW_\alpha)\bar{W}_{\dot\alpha}(\mathcal{D}_a\bar{W}^{\dot\alpha})+\\
&\qquad\qquad+(\mathcal{D}^a\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}(\mathcal{D}_aW_\alpha)-(\mathcal{D}^a\bar{W}_{\dot\alpha})W_\alpha(\mathcal{D}_a\bar{W}^{\dot\alpha})\Big)+\\
&\\
&\quad+44(\mathcal{D}^a\mathcal{D}^bW^\alpha)\Big((\mathcal{D}_a\mathcal{D}_bW_\alpha)\bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}+(\mathcal{D}_a\mathcal{D}_b\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}W_\alpha-(\mathcal{D}_a\mathcal{D}_b\bar{W}_{\dot\alpha})W_\alpha\bar{W}^{\dot\alpha}\Big)+\\
&\quad+12(\mathcal{D}^a\mathcal{D}^bW^\alpha)\Big(W_\alpha(\mathcal{D}_a\mathcal{D}_b\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}+\bar{W}_{\dot\alpha}(\mathcal{D}_a\mathcal{D}_b\bar{W}^{\dot\alpha})W_\alpha-\bar{W}_{\dot\alpha}(\mathcal{D}_a\mathcal{D}_bW_\alpha)\bar{W}^{\dot\alpha}\Big)+\\
&\quad+52(\mathcal{D}^a\mathcal{D}^bW^\alpha)\Big((\mathcal{D}_aW_\alpha)(\mathcal{D}_b\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}+\\
&\qquad\qquad+(\mathcal{D}_a\bar{W}_{\dot\alpha})(\mathcal{D}_b\bar{W}^{\dot\alpha})W_\alpha-(\mathcal{D}_a\bar{W}_{\dot\alpha})(\mathcal{D}_bW_\alpha)\bar{W}^{\dot\alpha}\Big)+
\end{aligned}$$

$$
\begin{aligned}
&+\ 52(\mathcal{D}^a\mathcal{D}^bW^\alpha)\Big(W_\alpha(\mathcal{D}_a\bar{W}_{\dot\alpha})(\mathcal{D}_b\bar{W}^{\dot\alpha})\ +\\
&\qquad\qquad +\ \bar{W}_{\dot\alpha}(\mathcal{D}_a\bar{W}^{\dot\alpha})(\mathcal{D}_bW_\alpha) - \bar{W}_{\dot\alpha}(\mathcal{D}_aW_\alpha)(\mathcal{D}_b\bar{W}^{\dot\alpha})\Big)\ +\\
&+\ 64(\mathcal{D}^a\mathcal{D}^bW^\alpha)\Big((\mathcal{D}_aW_\alpha)\bar{W}_{\dot\alpha}(\mathcal{D}_b\bar{W}^{\dot\alpha})\ +\\
&\qquad\qquad +\ (\mathcal{D}_a\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}(\mathcal{D}_bW_\alpha) - (\mathcal{D}_a\bar{W}_{\dot\alpha})W_\alpha(\mathcal{D}_b\bar{W}^{\dot\alpha})\Big)\ +\\
&+9(\mathcal{D}^aW^\alpha)\Big((\mathcal{D}^bW_\alpha)(\mathcal{D}_a\bar{W}_{\dot\alpha})(\mathcal{D}_b\bar{W}^{\dot\alpha})\ +\\
&\qquad\qquad +\ (\mathcal{D}^b\bar{W}_{\dot\alpha})(\mathcal{D}_a\bar{W}^{\dot\alpha})(\mathcal{D}_bW_\alpha) - (\mathcal{D}^b\bar{W}_{\dot\alpha})(\mathcal{D}_aW_\alpha)(\mathcal{D}_b\bar{W}^{\dot\alpha})\Big)\ +\\
&+\ 26(\mathcal{D}^aW^\alpha)\Big((\mathcal{D}_aW_\alpha)(\mathcal{D}^b\bar{W}_{\dot\alpha})(\mathcal{D}_b\bar{W}^{\dot\alpha})\ +\\
&\qquad\qquad +\ (\mathcal{D}_a\bar{W}_{\dot\alpha})(\mathcal{D}^b\bar{W}^{\dot\alpha})(\mathcal{D}_bW_\alpha) - (\mathcal{D}_a\bar{W}_{\dot\alpha})(\mathcal{D}^bW_\alpha)(\mathcal{D}_b\bar{W}^{\dot\alpha})\Big)\ +\\
&+\ 84G^{ab}(\mathcal{D}_aW^\alpha)\Big((\mathcal{D}_bW_\alpha)\bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}+(\mathcal{D}_b\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}W_\alpha-(\mathcal{D}_b\bar{W}_{\dot\alpha})W_\alpha\bar{W}^{\dot\alpha}\Big)\ +\\
&+\ 84G^{ab}(\mathcal{D}_aW^\alpha)\Big(W_\alpha\bar{W}_{\dot\alpha}(\mathcal{D}_b\bar{W}^{\dot\alpha})+\bar{W}_{\dot\alpha}\bar{W}^{\dot\alpha}(\mathcal{D}_bW_\alpha)-\bar{W}_{\dot\alpha}W_\alpha(\mathcal{D}_b\bar{W}^{\dot\alpha})\Big)\ +\\
&+\ 60G^{ab}(\mathcal{D}_aW^\alpha)\Big(W_\alpha(\mathcal{D}_b\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}+\bar{W}_{\dot\alpha}(\mathcal{D}_b\bar{W}^{\dot\alpha})W_\alpha-\bar{W}_{\dot\alpha}(\mathcal{D}_bW_\alpha)\bar{W}^{\dot\alpha}\Big)\ +\\
&-4G^{ab}W^\alpha\Big((\mathcal{D}_aW_\alpha)(\mathcal{D}_b\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}+(\mathcal{D}_a\bar{W}_{\dot\alpha})(\mathcal{D}_b\bar{W}^{\dot\alpha})W_\alpha-(\mathcal{D}_a\bar{W}_{\dot\alpha})(\mathcal{D}_bW_\alpha)\bar{W}^{\dot\alpha}\Big)\ +\\
&-\ 4G^{ab}W^\alpha\Big((\mathcal{D}_aW_\alpha)\bar{W}_{\dot\alpha}(\mathcal{D}_b\bar{W}^{\dot\alpha})+(\mathcal{D}_a\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}(\mathcal{D}_bW_\alpha)-(\mathcal{D}_a\bar{W}_{\dot\alpha})W_\alpha(\mathcal{D}_b\bar{W}^{\dot\alpha})\Big)\ +\\
&+32G^{ab}W^\alpha\Big(W_\alpha(\mathcal{D}_a\bar{W}_{\dot\alpha})(\mathcal{D}_b\bar{W}^{\dot\alpha})+\bar{W}_{\dot\alpha}(\mathcal{D}_a\bar{W}^{\dot\alpha})(\mathcal{D}_bW_\alpha)-\bar{W}_{\dot\alpha}(\mathcal{D}_aW_\alpha)(\mathcal{D}_b\bar{W}^{\dot\alpha})\Big)+\\
&+\ (\mathcal{D}^a\mathcal{D}_a\mathcal{D}_\alpha W^\beta)\Big(16W^\alpha W_\beta\bar{W}^2+18\bar{W}_{\dot\alpha}W^\alpha W_\beta\bar{W}^{\dot\alpha}+16\bar{W}^2W^\alpha W_\beta\ +\\
&\qquad\qquad +\ 50W^\alpha\bar{W}^2W_\beta-34W^\alpha\bar{W}_{\dot\alpha}W_\beta\bar{W}^{\dot\alpha}-34\bar{W}_{\dot\alpha}W^\alpha\bar{W}^{\dot\alpha}W_\beta\Big)\ +\\
&+\ (\mathcal{D}^a\mathcal{D}_\alpha W^\beta)\Big(16(\mathcal{D}_aW^\alpha)W_\beta\bar{W}^2+24(\mathcal{D}_a\bar{W}_{\dot\alpha})W^\alpha W_\beta\bar{W}^{\dot\alpha}\ +\\
&\qquad\qquad +\ 24(\mathcal{D}_a\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}W^\alpha W_\beta+64(\mathcal{D}_aW^\alpha)\bar{W}^2W_\beta\ -\\
&\qquad\qquad -\ 40(\mathcal{D}_aW^\alpha)\bar{W}_{\dot\alpha}W_\beta\bar{W}^{\dot\alpha}-48(\mathcal{D}_a\bar{W}_{\dot\alpha})W^\alpha\bar{W}^{\dot\alpha}W_\beta\Big)\ +\\
&+\ (\mathcal{D}^a\mathcal{D}_\alpha W^\beta)\Big(8W^\alpha(\mathcal{D}_aW_\beta)\bar{W}^2+12\bar{W}_{\dot\alpha}(\mathcal{D}_aW^\alpha)W_\beta\bar{W}^{\dot\alpha}\ +\\
&\qquad\qquad +\ 16\bar{W}_{\dot\alpha}(\mathcal{D}_a\bar{W}^{\dot\alpha})W^\alpha W_\beta+36W^\alpha(\mathcal{D}_a\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}W_\beta\ -\\
&\qquad\qquad -\ 20W^\alpha(\mathcal{D}_a\bar{W}_{\dot\alpha})W_\beta\bar{W}^{\dot\alpha}-28\bar{W}_{\dot\alpha}(\mathcal{D}_aW^\alpha)\bar{W}^{\dot\alpha}W_\beta\Big)\ +\\
&+\ (\mathcal{D}^a\mathcal{D}_\alpha W^\beta)\Big(16W^\alpha W_\beta(\mathcal{D}_a\bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}+12\bar{W}_{\dot\alpha}W^\alpha(\mathcal{D}_aW_\beta)\bar{W}^{\dot\alpha}\ +\\
&\qquad\qquad +\ 8\bar{W}^2(\mathcal{D}_aW^\alpha)W_\beta+36W^\alpha\bar{W}_{\dot\alpha}(\mathcal{D}_a\bar{W}^{\dot\alpha})W_\beta\ -\\
&\qquad\qquad -\ 28W^\alpha\bar{W}_{\dot\alpha}(\mathcal{D}_aW_\beta)\bar{W}^{\dot\alpha}-20\bar{W}_{\dot\alpha}W^\alpha(\mathcal{D}_a\bar{W}^{\dot\alpha})W_\beta\Big)\ +\\
&+\ (\mathcal{D}^a\mathcal{D}_\alpha W^\beta)\Big(24W^\alpha W_\beta\bar{W}_{\dot\alpha}(\mathcal{D}_a\bar{W}^{\dot\alpha})+24\bar{W}_{\dot\alpha}W^\alpha W_\beta(\mathcal{D}_a\bar{W}^{\dot\alpha})\ +\\
&\qquad\qquad +\ 16\bar{W}^2W^\alpha(\mathcal{D}_aW_\beta)+64W^\alpha\bar{W}^2(\mathcal{D}_aW_\beta)\ -\\
&\qquad\qquad -\ 48W^\alpha\bar{W}_{\dot\alpha}W_\beta(\mathcal{D}_a\bar{W}^{\dot\alpha})-40\bar{W}_{\dot\alpha}W^\alpha\bar{W}^{\dot\alpha}(\mathcal{D}_aW_\beta)\Big)\ +
\end{aligned}
$$

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$$
\begin{aligned}
&+ (\mathcal{D}_\alpha W^\beta)\Big(10(\mathcal{D}^a\mathcal{D}_a W^\alpha)W_\beta \bar{W}^2 + 16(\mathcal{D}^a\mathcal{D}_a \bar{W}_{\dot\alpha})W^\alpha W_\beta \bar{W}^{\dot\alpha} + \\
&\qquad + 18(\mathcal{D}^a\mathcal{D}_a \bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha} W^\alpha W_\beta + 44(\mathcal{D}^a\mathcal{D}_a W^\alpha)\bar{W}^2 W_\beta - \\
&\qquad - 26(\mathcal{D}^a\mathcal{D}_a W^\alpha)\bar{W}_{\dot\alpha} W_\beta \bar{W}^{\dot\alpha} - 34(\mathcal{D}^a\mathcal{D}_a \bar{W}_{\dot\alpha})W^\alpha \bar{W}^{\dot\alpha} W_\beta\Big) + \\
&+ (\mathcal{D}_\alpha W^\beta)\Big(10W^\alpha(\mathcal{D}^a\mathcal{D}_a W_\beta)\bar{W}^2 + 10\bar{W}_{\dot\alpha}(\mathcal{D}^a\mathcal{D}_a W^\alpha)W_\beta \bar{W}^{\dot\alpha} + \\
&\qquad + 16\bar{W}_{\dot\alpha}(\mathcal{D}^a\mathcal{D}_a \bar{W}^{\dot\alpha})W^\alpha W_\beta + 36W^\alpha(\mathcal{D}^a\mathcal{D}_a \bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha} W_\beta - \\
&\qquad - 20W^\alpha(\mathcal{D}^a\mathcal{D}_a \bar{W}_{\dot\alpha})W_\beta \bar{W}^{\dot\alpha} - 26\bar{W}_{\dot\alpha}(\mathcal{D}^a\mathcal{D}_a W^\alpha)\bar{W}^{\dot\alpha} W_\beta\Big) + \\
&+ (\mathcal{D}_\alpha W^\beta)\Big(16W^\alpha W_\beta(\mathcal{D}^a\mathcal{D}_a \bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha} + 10\bar{W}_{\dot\alpha} W^\alpha(\mathcal{D}^a\mathcal{D}_a W_\beta)\bar{W}^{\dot\alpha} + \\
&\qquad + 10\bar{W}^2(\mathcal{D}^a\mathcal{D}_a W^\alpha)W_\beta + 36W^\alpha \bar{W}_{\dot\alpha}(\mathcal{D}^a\mathcal{D}_a \bar{W}^{\dot\alpha})W_\beta - \\
&\qquad - 26W^\alpha \bar{W}_{\dot\alpha}(\mathcal{D}^a\mathcal{D}_a W_\beta)\bar{W}^{\dot\alpha} - 20\bar{W}_{\dot\alpha} W^\alpha(\mathcal{D}^a\mathcal{D}_a \bar{W}^{\dot\alpha})W_\beta\Big) + \\
&+ (\mathcal{D}_\alpha W^\beta)\Big(18W^\alpha W_\beta \bar{W}_{\dot\alpha}(\mathcal{D}^a\mathcal{D}_a \bar{W}^{\dot\alpha}) + 16\bar{W}_{\dot\alpha} W^\alpha W_\beta(\mathcal{D}^a\mathcal{D}_a \bar{W}^{\dot\alpha}) + \\
&\qquad + 10\bar{W}^2 W^\alpha(\mathcal{D}^a\mathcal{D}_a W_\beta) + 44W^\alpha \bar{W}^2(\mathcal{D}^a\mathcal{D}_a W_\beta) - \\
&\qquad - 34W^\alpha \bar{W}_{\dot\alpha} W_\beta(\mathcal{D}^a\mathcal{D}_a \bar{W}^{\dot\alpha}) - 26\bar{W}_{\dot\alpha} W^\alpha \bar{W}^{\dot\alpha}(\mathcal{D}^a\mathcal{D}_a W_\beta)\Big) + \\
&+ (\mathcal{D}_\alpha W^\beta)\Big(4(\mathcal{D}^a W^\alpha)(\mathcal{D}_a W_\beta)\bar{W}^2 + 16(\mathcal{D}^a \bar{W}_{\dot\alpha})(\mathcal{D}_a W^\alpha)W_\beta \bar{W}^{\dot\alpha} + \\
&\qquad + 24(\mathcal{D}^a \bar{W}_{\dot\alpha})(\mathcal{D}_a \bar{W}^{\dot\alpha})W^\alpha W_\beta + 44(\mathcal{D}^a W^\alpha)(\mathcal{D}_a \bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha} W_\beta - \\
&\qquad - 20(\mathcal{D}^a W^\alpha)(\mathcal{D}_a \bar{W}_{\dot\alpha})W_\beta \bar{W}^{\dot\alpha} - 40(\mathcal{D}^a \bar{W}_{\dot\alpha})(\mathcal{D}_a W^\alpha)\bar{W}^{\dot\alpha} W_\beta\Big) + \\
&+ (\mathcal{D}_\alpha W^\beta)\Big(8(\mathcal{D}^a W^\alpha)W_\beta(\mathcal{D}_a \bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha} + 8(\mathcal{D}^a \bar{W}_{\dot\alpha})W^\alpha(\mathcal{D}_a W_\beta)\bar{W}^{\dot\alpha} + \\
&\qquad + 12(\mathcal{D}^a \bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}(\mathcal{D}_a W^\alpha)W_\beta + 28(\mathcal{D}^a W^\alpha)\bar{W}_{\dot\alpha}(\mathcal{D}_a \bar{W}^{\dot\alpha})W_\beta - \\
&\qquad - 16(\mathcal{D}^a W^\alpha)\bar{W}_{\dot\alpha}(\mathcal{D}_a W_\beta)\bar{W}^{\dot\alpha} - 20(\mathcal{D}^a \bar{W}_{\dot\alpha})W^\alpha(\mathcal{D}_a \bar{W}^{\dot\alpha})W_\beta\Big) + \\
&+ (\mathcal{D}_\alpha W^\beta)\Big(12(\mathcal{D}^a W^\alpha)W_\beta \bar{W}_{\dot\alpha}(\mathcal{D}_a \bar{W}^{\dot\alpha}) + 16(\mathcal{D}^a \bar{W}_{\dot\alpha})W^\alpha W_\beta(\mathcal{D}_a \bar{W}^{\dot\alpha}) + \\
&\qquad + 12(\mathcal{D}^a \bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha} W^\alpha(\mathcal{D}_a W_\beta) + 40(\mathcal{D}^a W^\alpha)\bar{W}^2(\mathcal{D}_a W_\beta) - \\
&\qquad - 28(\mathcal{D}^a W^\alpha)\bar{W}_{\dot\alpha} W_\beta(\mathcal{D}_a \bar{W}^{\dot\alpha}) - 28(\mathcal{D}^a \bar{W}_{\dot\alpha})W^\alpha \bar{W}^{\dot\alpha}(\mathcal{D}_a W_\beta)\Big) + \\
&+ (\mathcal{D}_\alpha W^\beta)\Big(16W^\alpha(\mathcal{D}^a W_\beta)(\mathcal{D}_a \bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha} + 4\bar{W}_{\dot\alpha}(\mathcal{D}^a W^\alpha)(\mathcal{D}_a W_\beta)\bar{W}^{\dot\alpha} + \\
&\qquad + 16\bar{W}_{\dot\alpha}(\mathcal{D}^a \bar{W}^{\dot\alpha})(\mathcal{D}_a W^\alpha)W_\beta + 36W^\alpha(\mathcal{D}^a \bar{W}_{\dot\alpha})(\mathcal{D}_a \bar{W}^{\dot\alpha})W_\beta - \\
&\qquad - 20W^\alpha(\mathcal{D}^a \bar{W}_{\dot\alpha})(\mathcal{D}_a W_\beta)\bar{W}^{\dot\alpha} - 20\bar{W}_{\dot\alpha}(\mathcal{D}^a W^\alpha)(\mathcal{D}_a \bar{W}^{\dot\alpha})W_\beta\Big) + \\
&+ (\mathcal{D}_\alpha W^\beta)\Big(12W^\alpha(\mathcal{D}^a W_\beta)\bar{W}_{\dot\alpha}(\mathcal{D}_a \bar{W}^{\dot\alpha}) + 8\bar{W}_{\dot\alpha}(\mathcal{D}^a W^\alpha)W_\beta(\mathcal{D}_a \bar{W}^{\dot\alpha}) + \\
&\qquad + 8\bar{W}_{\dot\alpha}(\mathcal{D}^a \bar{W}^{\dot\alpha})W^\alpha(\mathcal{D}_a W_\beta) + 28W^\alpha(\mathcal{D}^a \bar{W}_{\dot\alpha})\bar{W}^{\dot\alpha}(\mathcal{D}_a W_\beta) - \\
&\qquad - 20W^\alpha(\mathcal{D}^a \bar{W}_{\dot\alpha})W_\beta(\mathcal{D}_a \bar{W}^{\dot\alpha}) - 16\bar{W}_{\dot\alpha}(\mathcal{D}^a W^\alpha)\bar{W}^{\dot\alpha}(\mathcal{D}_a W_\beta)\Big) + \\
&+ (\mathcal{D}_\alpha W^\beta)\Big(24W^\alpha W_\beta(\mathcal{D}^a \bar{W}_{\dot\alpha})(\mathcal{D}_a \bar{W}^{\dot\alpha}) + 16\bar{W}_{\dot\alpha} W^\alpha(\mathcal{D}^a W_\beta)(\mathcal{D}_a \bar{W}^{\dot\alpha}) + \\
&\qquad + 4\bar{W}^2(\mathcal{D}^a W^\alpha)(\mathcal{D}_a W_\beta) + 44W^\alpha \bar{W}_{\dot\alpha}(\mathcal{D}^a \bar{W}^{\dot\alpha})(\mathcal{D}_a W_\beta) - \\
&\qquad - 40W^\alpha \bar{W}_{\dot\alpha}(\mathcal{D}^a W_\beta)(\mathcal{D}_a \bar{W}^{\dot\alpha}) - 20\bar{W}_{\dot\alpha} W^\alpha(\mathcal{D}^a \bar{W}^{\dot\alpha})(\mathcal{D}_a W_\beta)\Big) +
\end{aligned}
$$

$$
\begin{aligned}
&+\,14(\mathcal{D}_\alpha W^\beta)(\bar{\mathcal{D}}_{\dot\alpha}\bar W_{\dot\beta})\Big(W^\alpha\bar W^{\dot\alpha}\bar W^{\dot\beta}W_\beta+\bar W^{\dot\alpha}W^\alpha W_\beta\bar W^{\dot\beta}-\\
&\qquad\qquad -\,W^\alpha\bar W^{\dot\alpha}W_\beta\bar W^{\dot\beta}-\bar W^{\dot\alpha}W^\alpha\bar W^{\dot\beta}W_\beta\Big)+\\
&+\,14(\mathcal{D}_\alpha W^\beta)\Big(W^\alpha(\bar{\mathcal{D}}_{\dot\alpha}\bar W_{\dot\beta})\bar W^{\dot\alpha}\bar W^{\dot\beta}W_\beta+\bar W^{\dot\alpha}(\bar{\mathcal{D}}_{\dot\alpha}\bar W_{\dot\beta})W^\alpha\bar W^{\dot\beta}W_\beta-\\
&\qquad -\,W^\alpha(\bar{\mathcal{D}}_{\dot\alpha}\bar W_{\dot\beta})\bar W^{\dot\alpha}W_\beta\bar W^{\dot\beta}-\bar W^{\dot\alpha}(\bar{\mathcal{D}}_{\dot\alpha}\bar W_{\dot\beta})\bar W^{\dot\beta}W^\alpha W_\beta\Big)+\\
&+\,14(\mathcal{D}_\alpha W^\beta)\Big(\bar W^{\dot\alpha}W^\alpha(\bar{\mathcal{D}}_{\dot\alpha}\bar W_{\dot\beta})\bar W^{\dot\beta}W_\beta-W^\alpha\bar W^{\dot\alpha}(\bar{\mathcal{D}}_{\dot\alpha}\bar W_{\dot\beta})\bar W^{\dot\beta}W_\beta\Big)+\\
&+\,7(\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\alpha W^\beta)W^\alpha\Big(2\bar W_{\dot\beta}\bar W^{\dot\alpha}W_\beta\bar W^{\dot\beta}+\bar W_{\dot\beta}\bar W^{\dot\alpha}\bar W^{\dot\beta}W_\beta+W_\beta\bar W^{\dot\alpha}\bar W^2-\\
&\qquad\qquad -\,2\bar W^{\dot\alpha}\bar W_{\dot\beta}W_\beta\bar W^{\dot\beta}-2\bar W^{\dot\alpha}\bar W^2W_\beta-\bar W_{\dot\beta}W_\beta\bar W_{\dot\alpha}\bar W^{\dot\beta}\Big)+\\
&+\,7(\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\alpha W^\beta)\bar W^{\dot\alpha}\Big(6W^\alpha\bar W_{\dot\beta}W_\beta\bar W^{\dot\beta}+\bar W^2W^\alpha W_\beta-3W^\alpha W_\beta\bar W^2-\\
&\qquad\qquad -\,3\bar W_{\dot\beta}W^\alpha W_\beta\bar W^{\dot\beta}-2W^\alpha\bar W^2W_\beta\Big)+\\
&+\,7(\bar{\mathcal{D}}_{\dot\alpha}\mathcal{D}_\alpha W^\beta)\bar W_{\dot\beta}\Big(2\bar W^{\dot\alpha}W^\alpha W_\beta\bar W^{\dot\beta}+W^\alpha W_\beta\bar W^{\dot\alpha}\bar W^{\dot\beta}+\bar W^{\dot\alpha}W^\alpha\bar W^{\dot\beta}W_\beta-\\
&\qquad\qquad -\,3W^\alpha\bar W^{\dot\alpha}W_\beta\bar W^{\dot\beta}-\bar W^{\dot\alpha}\bar W^{\dot\beta}W^\alpha W_\beta\Big)+\\
&+\,(\mathcal{D}_\alpha W^\beta)(\mathcal{D}_\beta W^\alpha)\Big(5W^2\bar W^2+5\bar W_{\dot\alpha}W^2\bar W^{\dot\alpha}+5\bar W^2W^2+\\
&\qquad\qquad +\,33W^\gamma\bar W^2W_\gamma-19W^\gamma\bar W_{\dot\alpha}W_\gamma\bar W^{\dot\alpha}-19\bar W_{\dot\alpha}W^\gamma\bar W^{\dot\alpha}W_\gamma\Big)+\\
&+\,(\mathcal{D}_\alpha W^\beta)\Big(6W^\gamma(\mathcal{D}_\beta W^\alpha)\bar W_{\dot\alpha}W_\gamma\bar W^{\dot\alpha}+8\bar W_{\dot\alpha}(\mathcal{D}_\beta W^\alpha)W^2\bar W^{\dot\alpha}+\\
&\qquad +\,8\bar W_{\dot\alpha}(\mathcal{D}_\beta W^\alpha)\bar W^{\dot\alpha}W^2-6W^\gamma(\mathcal{D}_\beta W^\alpha)W_\gamma\bar W^2-\\
&\qquad -\,6W^\gamma(\mathcal{D}_\beta W^\alpha)\bar W^2W_\gamma-22\bar W_{\dot\alpha}(\mathcal{D}_\beta W^\alpha)W^\gamma\bar W^{\dot\alpha}W_\gamma\Big)+\\
&+\,(\mathcal{D}_\alpha W^\beta)\Big(9W^2(\mathcal{D}_\beta W^\alpha)\bar W^2+\frac52W^\gamma\bar W_{\dot\alpha}(\mathcal{D}_\beta W^\alpha)W_\gamma\bar W^{\dot\alpha}+\\
&\qquad +\,\frac52\bar W_{\dot\alpha}W^\gamma(\mathcal{D}_\beta W^\alpha)\bar W^{\dot\alpha}W_\gamma-5W^\gamma\bar W_{\dot\alpha}(\mathcal{D}_\beta W^\alpha)\bar W^{\dot\alpha}W_\gamma\Big)+\\
&+\,(\mathcal{D}_\alpha W^\beta)(\mathcal{D}_\gamma W^\alpha)\Big(14W_\beta W^\gamma\bar W^2+14\bar W_{\dot\alpha}W_\beta W^\gamma\bar W^{\dot\alpha}+14\bar W^2W_\beta W^\gamma+\\
&\qquad\qquad +\,42W_\beta\bar W^2W^\gamma-28W_\beta\bar W_{\dot\alpha}W^\gamma\bar W^{\dot\alpha}-28\bar W_{\dot\alpha}W_\beta\bar W^{\dot\alpha}W^\gamma\Big)+\\
&+\,14(\mathcal{D}_\alpha W^\beta)\bar W_{\dot\alpha}(\mathcal{D}_\gamma W^\alpha)\Big(W_\beta W^\gamma\bar W^{\dot\alpha}+\bar W^{\dot\alpha}W_\beta W^\gamma-2W_\beta\bar W^{\dot\alpha}W^\gamma\Big)+\\
&+\,14(\mathcal{D}_\alpha W^\beta)\bar W^2(\mathcal{D}_\gamma W^\alpha)W_\beta W^\gamma\Big)+\text{c.c.}\,.
\end{aligned}
$$

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